Lexicographic least-squares (reliable solution and applications in robotics)

This project will address the reliable (and efficient) solution of a *lexicographic least-squares* problem and its applications to control of robotic systems. This is a multi-objective optimization problem of the form

$$\underset{x,\, v_1,\ldots,v_p}{\text{lex minimize}} \quad f_0 = (\|v_1\|^2,\, \ldots,\, \|v_p\|^2)$$

$$\text{subject to} \quad \begin{bmatrix} b_1^l \\ \vdots \\ b_p^l \end{bmatrix} \leq \begin{bmatrix} A_1 \\ \vdots \\ A_p \end{bmatrix} x - \begin{bmatrix} v_1 \\ \vdots \\ v_p \end{bmatrix} \leq \begin{bmatrix} b_1^u \\ \vdots \\ b_p^u \end{bmatrix},$$

where $b_k^l \leq A_k x \leq b_k^u$ defines a polyhedral constraint, whose violation v_k is to be minimized in a least-squares sense. "lex minimize" means that the objective function $f_0 \in \mathbb{R}^p$ is to be optimized lexicographically. That is, minimizing $\|v_k\|^2$ is infinitely more important that minimizing $\|v_j\|^2$ for $k < j$. This can be seen as a strict hierarchy of constraints with different priorities and has many interesting applications [1]. We have already developed a solver that can be used by the student.

Two problems will be dealt with during the project:

1. Application of lexicographic optimization in the context of inverse kinematics control of under-actuated robots. This would require formulating appropriate optimization problems that reflect given design specifications, as well as reliably solving these problems in the context of real-time control of a humanoid robot. For example, techniques like Model Predictive Control would be used.

2. The reliable solutions of the above problem requires the development and comparison of various regularization techniques. This is necessary in order to deal with ill-conditioned problems often occurring when a robot is close to a singular configuration.

- The project could lead to a thesis.

- Prerequisites: good familiarity with basic principles in optimization, basic coding skills in Matlab and `C++`, basic knowledge of control theory would help.

- Place: Inria (BIPOP team).

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References

[1] A. Escande, N. Mansard, and P.-B. Wieber, "Hierarchical quadratic programming: Fast online humanoid-robot motion generation," *The International Journal of Robotics Research*, 33 (7), pp. 1006-1028, 2014.