Master thesis project "Uncertainty quantification in Stochastic Differential Equations and applications to Neurosciences"

Mention: Research

Advisors:

Clémentine Prieur, Clementine.Prieur@univ-grenoble-alpes.fr, 04 57 42 17 77

Pierre Étoré, Pierre.Etore@univ-grenoble-alpes.fr, 04 57 42 17 35

Laboratory / working place: LJK, IMAG building, 700 avenue centrale, Campus de Saint Martin d'Hères, 38401 Domaine Universitaire de Saint-Martin-d'Hères

Phd forseen: A PhD could be forseen, the subject is conceived so.

Project objectives and required competences:

Many mathematical models involve input parameters, which are not precisely known. Global sensitivity analysis aims at identifying the parameters whose uncertainty has the largest impact on the variability of a quantity of interest - for instance by computing Sobol' sensitivity indices.

In this project we consider stochastic models described by stochastic differential equations (SDE), whose coefficients depend on some uncertainty parameter ξ, i.e. SDE of the form

$$dX_t = \sigma(\xi, X_t)dW_t + b(\xi, X_t)dt.$$

We assume that ξ is independent from the Wiener process W. We focus the study on mean quantities, defined as the expectation with respect to the law of W of a quantity of interest related to the solution X of the SDE. These mean quantities depend on ξ. To handle this kind of problem, very few references exist. One can mention [LMK15] and [EPPL18]. In this latter work the approach is based on a Feynman-Kac representation of the quantity of interest, from which one gets a parametrized partial differential equation (PDE) representation of the initial problem. Then one can handle the uncertainty on the parametrized PDE using existing methods involving some polynomial chaos expansion and some stochastic Galerkin projection (see [Nou09]). Taking the sequel of the study led in [EPPL18] we would like to handle more complex SDE, such as the Morris-Lecar or the Fitzhugh-Nagumo models, arising form neurosciences (see for instance [DS14]).

We seek for a student in probability and statistics with some knowledge and/or interest in PDE issues. Some knowledge of scientific computing is required (C/C++, Matlab, R or Python).

References

[DS14] Susanne Ditlevsen and Adeline Samson, *Parameter estimation in the stochastic morris-lecar neuronal model with particle filter methods*, Annals of Applied Statistics, Vol. 8, No. 2, 2014, pp 674-702.

[EPPL18] Pierre Etoré, Clémentine Prieur, Dang Khoi Pham and Long Li, *Global sensitivity analysis for models described by stochastic differential equations*, Methodology and Computing in Applied Probability, published online july 2019, `https://hal.archives-ouvertes.fr/hal-01926919/document`

[LMK15] O.P. Le Maître, and O.M. Knio. *PC analysis of stochastic differential equations driven by Wiener noise*, Reliability Engineering and System Safety, 2015, pp 107-124.

[Nou09] Anthony Nouy, *Recent developments in spectral stochastic methods for the numerical solution of stochastic partial differential equations*, Archives of Computational Methods in Engineering- 16(3), 2009, pp 251-285.