Numerical modelling of tight knots
MSc Research Internship 2020

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Hosting Laboratory: ELAN team (INRIA and LJK, Grenoble), https://team.inria.fr/elan/

Practical details: Paid internship (INRIA scale), possibility to continue with a PhD on a similar topic.

Context: Knots are fascinating and widely spread structures which combine the seeming simplicity of a thread with some intricate topology to obtain many different properties. As such, they have been mathematically studied since the 18^{th} century and *knot theory* closely developed along the early foundations of mathematical topology until the modern use of algebraic topology, which eventually allowed to classify knots using simple polynomial functions. However, while the geometry of knots is now well understood, their mechanical properties is still vague and the choice of a particular knot for some task is most of the time governed by empiricism. From the numerical perspective, some recent works have focused on reduced models for the simulation of threads [1, 7, 8, 2], thereby efficiently accounting for the non-linear behaviour of this slender structures. However, the challenging case of tight and self-locking knots, where both the knot topology and the thread mechanical behaviour come into play, have been poorly studied despite their intensive and crucial use in many fields such as surgery or rock-climbing, or their appearance in many biological phenomena [4].

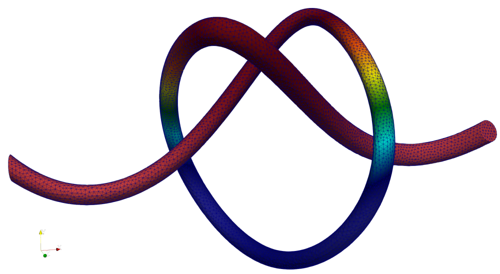

Figure 1: Overhand knot 3D model

Objectives: The goal of this project is to develop numerical models for the study of tight knots. To this end, various approaches will be considered, ranging from full ab-initio finite-element models to reduced physical models, and the numerical aspects will be built upon tools developed by researchers from the ELAN team [5, 3, 6].

Required skills: Good skills in numerical analysis (modelling, numerical discretisation of ODEs and PDEs, finite elements, optimisation) as well as in algorithmic and programmation (C/C++, Python) are required. Curiosity and taste for applications in mechanics and computer graphics would be appreciated.

References

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[3] R. Casati and F. Bertails-Descoubes. Super space clothoids. *ACM Transactions on Graphics (Proc. ACM SIGGRAPH'13)*, 32(4):48:1–48:12, July 2013.

[4] Nicolas Clauvelin. *Contact au sein des structures élancées : sur-enroulement de l'ADN et noeuds élastiques*. PhD thesis, 2008.

[5] G. Daviet, F. Bertails-Descoubes, and L. Boissieux. A hybrid iterative solver for robustly capturing Coulomb friction in hair dynamics. *ACM Transactions on Graphics (Proc. ACM SIGGRAPH Asia'11)*, 30:139:1–139:12, 2011.

[6] Christophe Prud'homme, Vincent Chabannes, and Thibaut Metivet. feelpp/feelpp: Feel++ v0.107, 2019.

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[8] Jonas Spillmann and Matthias Teschner. An adaptive contact model for the robust simulation of knots. In *Computer Graphics Forum*, volume 27, pages 497–506. Wiley Online Library, 2008.

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