The exponential X-Ray transform in fan-beam format

Rolf Clackdoyle <rolf.clackdoyle@univ-grenoble-alpes.fr>
Laurent Desbat <laurent.desbat@univ-grenoble-alpes.fr>
Simon Rit <simon.rit@creatis.insa-lyon.fr>

TIMC-IMAG, CNRS, Univ. Grenoble Alpes, Grenoble
CREATIS, CNRS, INSERM, INSA de Lyon, Lyon

Context. In two-dimensions, the X-Ray transform and the Radon transform refer to the same operator (although the notation differs slightly). The exponential X-Ray transform is defined as $p(\phi, s) = \int f(r\alpha_\phi + s\beta_\phi)e^{\mu_0 r}dr$ where μ_0 is a known constant, and where $\alpha_\phi = (\cos\phi, \sin\phi)$, $\beta_\phi = (-\sin\phi, \cos\phi)$. If $\mu_0 = 0$ then the usual X-Ray (or Radon) transform appears. Range conditions (also known as "data consistency conditions") are known for the exponential X-ray transform [1]. In principle, these conditions can be applied even if the data were provided in fan-beam format: $d(\lambda, \phi) = \int_0^\infty f(v_\lambda + l\alpha_\phi)e^{\mu_0 l}dl$, where v_λ parameterizes a suitable curve in the plane such that all lines intersecting the object are included in the fan-beam measurements $d(\lambda, \phi)$. However, unlike the parallel-case $p(\phi, s)$, if only a subset of fan-beam projections $d(\lambda, \phi)$ are available, then the range conditions are not known, because the conditions are formulated in a "parallel-beam" fashion. Even for the non-exponential case ($\mu_0 = 0$), range conditions tailored for fan-beam projections have only been established fairly recently [2].

The exponential X-Ray transform is the relevant transformation for Single Photon Emission Computed Tomography (SPECT) scanning, which occurs routinely in all major hospitals. Data consistency conditions can be applied to detect and identify parameters of non-ideal physical effects, such as patient movement during the scan [3,4].

Objectives. The primary objective of this project is to explore range conditions for a finite number of divergent (fan-beam), exponential projections. The specific context is pinhole imaging in SPECT, which arises both for clinical and pre-clinical scanners. This Masters project has both mathematical and numerical simulations aspects.

Practical Information. The duration of this Masters (or PFE) project is 6 months. The primary location is the TIMC laboratory (Grenoble). Strong mathematical skills and strong programming skills (e.g. Python, matlab, C++) are required, and a course in mathematics of tomography.

References.

[1] V.Aguilar, P.Kuchment. "Range conditions for the multi-dimensional exponential x-ray transform." *Inverse Problems* 11: 977-982, 1995.

[2] R.Clackdoyle. "Necessary and sufficient consistency conditions for fanbeam projections along a line." *IEEE Transactions on Nuclear Science* 60: 1560-1569, 2013.

[3] C.Mennessier, F.Noo, R.Clackdoyle, G.Bal, L.Desbat. "Attenuation correction in SPECT using consistency conditions for the exponential ray transform." *Physics in Medicine and Biology* 44: 2483-2510, 1999.

[4] R.Clackdoyle, S.Rit, J.Hoskovec, L.Desbat. "Fanbeam data consistency conditions for applications to motion detection." *The Third International Meeting on Image Formation in X-Ray Computed Tomography, Salt Lake City, USA. June 22-25, 2015*. pp.253-256, 2015.