

Delayed control for the Korteweg-de-Vries equation

Internship at Laboratoire J. Kuntzmann
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Description: The general context of this internship is the control theory of a particular class of partial differential equations, more precisely the Korteweg-de-Vries equation (see e.g. [2] for a recent tutorial). This equation is important for fluid mechanics with boundary control. The goal is to analyze the stability of the nonlinear Korteweg-de-Vries with delayed boundary conditions. This equation can result from a boundary control with a delay in the loop. The work [1] is relevant because it deals on exponential stability with constant delay for the nonlinear Korteweg-de-Vries equation. Lyapunov method has been employed, and numerical simulations were performed on the linearized model.

The first aim of this internship is to compare the decreasing rates given by Lyapunov inequality with the spectral decomposition of the linearized Korteweg-de-Vries in presence of (or without any) boundary delay. Numerical simulations can also be fruitful. Then time-varying delays could be introduced for both linearized and nonlinear KdV equation. The papers [5, 4, 6] should be studied for the theoretical aspects. The works [7, 3] could be considered for the numerical simulations. As a final objective, the network case could be foreseen.

This internship could be followed by a PhD e.g. focusing on theoretical stability aspects or numerical analysis.

References

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