Research project proposal (M2 thesis)
Numerical optimal transport and nonimaging optics

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Location : *Laboratory Jean Kuntzmann*, in Grenoble, but the student may go once or twice to Orsay.

Scientific context

Nonimaging optics is a field of optics where one is interested in the transfer of light energy between a source and a target. Unlike traditional optics, or imaging-optics, the goal is not to reproduce an image of the input light, but to design optical components - such as mirrors or lenses - that transfer a given source light to a prescribed target light. This problem has applications in car lighting, solar oven and in problems where one wants to reduce light pollution.

The light transfer equation arising in optics is in fact a Monge-Ampère equation and can be recast in some cases as an optimal transport problem [7]. It is the case for certain mirrors [7] or lenses [4] with source light that are either ponctual or parallel, when the target is a set of directions (and not located at a finite distance).

These last years, geometrical methods, called *semi-discrete*, have been shown to be very efficient to accurately solve optimal transport problems. They are based on convex optimization and computational geometry and allow solve large instances of optimal transport problems [3, 2]. More recently a Newton algorithm has been shown to be able to handle up to more 100 millions of points. The convergence of this algorithm was shown for general cost functions on manifolds [1] and also for triangulations [5]. This algorithm has been applied for some problems in nonimaging optics [4].

However, some of the problems arising in nonimaging optics do not correspond to optimal transport problems, but to more general equations called *generated jacobian* [6]. It is in particular the case when the target light is at a finite distance.

Objectives

The subject proposed here consists in the extension of semi-discrete methods to solve generated jacobian equations. Numerical tests from [4] suggest that one may be able to approximate solutions of generated jacobian equations by solutions of optimal transport problems. The goal of the project here is therefore to study the linearization of generated jacobian equations with respect to the dual variable in order to approximate them by optimal transport problems. The problem will be first considered in some problems arising in nonimaging optics and will eventually be extended in a more general setting.

This internship may lead to a PhD, with applications in optics or in other fields.

Références

[1] Jun Kitagawa, Quentin Mérigot, and Boris Thibert. Convergence of a newton algorithm for semi-discrete optimal transport. *Journal of the European Mathematical Society, accepted*, 2016.

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[3] Quentin Mérigot. A multiscale approach to optimal transport. *Computer Graphics Forum*, 30 :1583–1592, 2011.

[4] Quentin Mérigot, Jocelyn Meyron, and Boris Thibert. An algorithm for optimal transport between a simplex soup and a point cloud. *SIAM Journal on Imaging Sciences*, 11(2) :1363–1389, 2018.

[5] Quentin Mérigot, Jocelyn Meyron, and Boris Thibert. Light in power : A general and parameter-free algorithm for caustic design. *accepted to TOG (proceedings of Siggraph Asia)*, 2018.

[6] Neil S Trudinger. On the local theory of prescribed jacobian equations. *arXiv preprint arXiv :1211.4661*, 2012.

[7] X.J. Wang. On the design of a reflector antenna ii. *Calculus of Variations and Partial Differential Equations*, 20(3) :329–341, 2004.