Analysis and optimization of Schwarz coupling algorithms for discretized diffusion equations *(research)*

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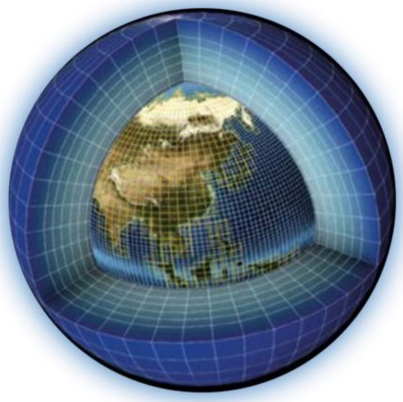
Context The interactions between atmosphere and ocean play a major role in many geophysical phenomena, covering a wide range of temporal scales (e.g. daily weather, tropical cyclones, global climate...). Therefore the numerical simulation of such phenomena requires coupled atmospheric and oceanic models, which properly represent the behavior of the boundary layers encompassing the air-sea interface and their two-way interactions.
A relevant mathematical model is to consider the coupling between two 1-D diffusion equations representing the turbulent mixing in the oceanic and atmospheric boundary layers. The boundary conditions at the ocean-atmosphere interface are computed through a so-called wall law which predicts a logarithmic profile for the solution in the vicinity of the interface. From a numerical point of view, such 1-D coupled systems have to be solved a huge number of times in actual simulations. It is thus of importance to derive efficient numerical methods for these calculations.
The Schwarz iterative coupling algorithms provide a good mathematical framework for this problem. Its convergence properties have been analyzed, as well as optimized strategies (Lemarié et al., 2013 ; Thery et al., 2018). However this work was performed working with the continuous formulation of the equations.

Objectives In this context, the objective of this work is to perform an analysis and an optimization of the Schwarz algorithm for this coupled problem at the discretized level. Following a strategy suggested in Zisowski and Ehrardt (2006) and Wu and Al-Khaleel (2017), several questions will be investigated : derivation of the convergence rate for semi-discretized equations, either in space or in time, or even for fully discretized equations ; optimization of this convergence rate w.r.t. free parameters ; comparison and consistency of these results w.r.t. those obtained with the continuous equations. Corresponding numerical simulations will be performed to confirm the theoretical results.



Prerequisites basic knowledge in numerical analysis, programming skills

References

Lemarié F., L. Debreu and E. Blayo, 2013 : Towards an optimized global-in-time Schwarz algorithm for diffusion equations with discontinuous and spatially variable coefficients. Part 2 : the variable coefficients case. *Elec. Trans. Num. Anal.*, **40**, 170-186.

Thery S., C. Pelletier, F. Lemarié and E. Blayo, 2018 : Analysis of Schwarz waveform relaxation for the coupled Ekman boundary layer problem with continuously variable coefficients. *In preparation*.

Wu S.L. and M.D. Al-Khaleel, 2017 : Optimized waveform relaxation methods for RC circuits : discrete case. *ESAIM : Mathematical Modelling and Numerical Analysis*, **51 (1)**, 209-223.

Zisowsky A. and M. Ehrhardt, 2006 : Discrete transparent boundary conditions for parabolic systems. *Mathematical and Computer Modelling*, **43**, 294-309.