Master's degree Research project

Tensor decomposition of Hyperspectral Images

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Location: GIPSA-Lab, Département Images et Signaux, 11 rue des Mathématiques, Saint Martin d'Hères

Duration: 6 months (about 500 euro/month)

Required competences: linear algebra, multivariate analysis, image and signal processing, Matlab or Python programming

Context Hyperspectral imagery is an imaging technique that simultaneously acquires up to several hundreds of images of a same scene at different spectral wavelengths. Each pixel is therefore no longer a triplet of values as it is the case in classical RGB imagery, but a n-dimensional vector corresponding to a reflectance spectrum. This allows a fine description of the spectral properties of the scene, leading to a better identification of physical materials composing it. Hyperspectral imagery has a number applications in various industrial and research fields such as remote sensing, chemistry, biology, biomedical engineering and astrophysics.

A common model for representing a hyperspectral image $\boldsymbol{X} \in \mathbb{R}^{N\times L}$ of N pixels and L bands is via a low-rank decomposition based on non-negative matrix factorization: $\boldsymbol{X} \approx \boldsymbol{\Phi}\boldsymbol{S}$ [1]. $\boldsymbol{S} \in \mathbb{R}_+^{R\times L}$ and $\boldsymbol{\Phi} \in \mathbb{R}_+^{N\times R}$ are two non-negative matrices representing respectively a set of R spectra of pure material (endmembers) and their proportions (abundances) for each pixel. The blind source separation problem of estimating the endmembers and their fractional abundances in hyperspectral images is commonly referred to as spectral unmixing [1].

When a third dimension is available (e.g., time), the acquisitions can be represented as a tensor (i.e., a three-way array) $\boldsymbol{\mathcal{X}}^{N\times L\times K}$ (being K the number of acquisitions such as time or angles) and can be effectively decomposed onto a few rank-one terms in a multilinear fashion via Canonical Polyadic (CP) decomposition [2]. CP has been applied recently for the decomposition of hyperspectral time series and multiangle acquisitions into low rank terms [3]. Formally, $\mathcal{X}_{ijk} \approx \sum_{r=1}^{R} A_{ij}B_{jr}C_{kr}$ in which the tensor $\boldsymbol{\mathcal{X}}$ is decomposed into a multilinear combination of non-negative factor matrices $\boldsymbol{A} \in \mathbb{R}_+^{N\times R}$, $\boldsymbol{B} \in \mathbb{R}_+^{L\times R}$ and $\boldsymbol{C} \in \mathbb{R}_+^{K\times R}$, of rank R that can be associated to the spatial, spectral and acquisition way, respectively.

Objectives of the internship This internship deals with the application of tensor decomposition approaches to a single hyperspectral image. A straightforward way to consider a hyperspectral data cube as a tensor relies on taking the two spatial dimensions i.e., rows and columns of the image, as two different ways. However, this representation is of little use for the CP decomposition since the resulting tensor is not low rank.

Recently, it was proposed to build a patch-tensor representation of a hyperspectral image by stacking small patches of the image as a fictive third way [4]. In this case, the augmentation of the data with the information given by patches provides indeed a low rank data structure (due to the self-similarity of the image content). The CP decomposition of this tensor allows to retrieve low rank terms that can be associated with endmembers (accounting also for their spectral variability) and their abundances.

This internship will focus more into the representation of a hyperspectral image as a tensor. In particular, alternative tensor representations will be studied. For example, a multiscale/multilevel decomposition of the image content (e.g., via wavelets [5] or mathematical morphological operators [6]) could be considered as well as a third way. Experiments will be conducted in order to compare different alternative approaches of representation. The results will be compared with respect to those obtained by more conventional non-negative matrix factorization such as in spectral unmixing. Synthetic and real images will be considered in the analysis.

[1] J. M. Bioucas-Dias, A. Plaza, N. Dobigeon, M. Parente, Q. Du, P. Gader, and J. Chanussot, "Hyperspectral unmixing overview: Geometrical, statistical, and sparse regression-based approaches," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 5, no. 2, pp. 354–379, 2012.

[2] P. Comon, "Tensors: A brief introduction," *IEEE Signal Processing Magazine*, vol. 31, no. 3, pp. 44–53, 2014.

[3] M. A. Veganzones, J. E. Cohen, R. C. Farias, J. Chanussot, and P. Comon, "Nonnegative tensor cp decomposition of hyperspectral data," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 54, no. 5, pp. 2577–2588, 2016.

[4] M. A. Veganzones, J. E. Cohen, R. C. Farias, K. Usevich, L. Drumetz, J. Chanussot, and P. Comon, "Canonical polyadic decomposition of hyperspectral patch tensors," in *EUSIPCO*, 2016.

[5] S. Mallat, *A wavelet tour of signal processing*. Academic press, 1999.

[6] P. Maragos, "Pattern spectrum and multiscale shape representation," *IEEE Transactions on pattern analysis and machine intelligence*, vol. 11, no. 7, pp. 701–716, 1989.