Dimension reduction and sensitivity analysis

Master 2 internship

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Context

Environmental numerical modeling is a crucial issue for numerical forecasting systems (meteorology, oceanography, atmospheric chemistry, rivers and floods, climate, etc.). Numerical environmental models are based on mathematical equations representing the physics of the system under study. In these models, many uncertainties are present:

- the equations are imperfect modeling of reality;
- they often involve poorly known parameters (initial and boundary conditions, coefficients in the equations, etc.);
- the numerical solution is a continuous approximation of the truncated solution at a certain spatial, temporal and/or frequency resolution.

Sensitivity analysis

It is important to measure the impact of a parameter on the quality of the solution and identify "sensitive" parameters in the sense that a small change in this parameter will induce a wide variation in the solution of the model. Sensitivity analysis is the set of stochastic or deterministic methods, which measure the sensitivity of a parameter in the model. Among the different methods of sensitivity analysis, we will focus on the probabilistic approach based on the computation of Sobol' sensitivity indices [5, 4]. Numerical computation of these indices requires many evaluations of the model [3, 6]. In many contexts, each run of the model takes a very long time, which makes it impossible to estimate Sobol' indices with the desired accuracy. It is then necessary to accelerate the evaluation of the model. The proposed approach is to reduce the dimension of the problem.

Dimension reduction

The idea of model reduction is as follows. When working with a non reduced model, we first discretize the (spatio-temporal) domain, and then solve the equations on the selected grid, that is to say that one seeks the values of the unknown variables in each point of the grid. If the grid has hundreds of thousands of discrete points, many variables have thus to be determined. Dimension reduction aims at reducing the number of variables to be computed, by using a well-chosen basis. The choice of the basis is critical and the effectiveness of the model depends on it.

The general principle is the following. Suppose that our system is described by a vector state $u \in E$ (for example, u contains the value of current velocities and the height of ocean water at each point of the domain), so that the dimension of u is high: it is the number of grid points multiplied by the number of variables (e.g. 10^6 in oceanography). The goal is to find well-suited vectors $(\phi_1, \phi_2, ..., \phi_r) \in E^r$, so that u can be decomposed with a good accuracy in this basis:

$$u \simeq \sum_{i=1}^{r} u_i \phi_i$$

where the u_i are real coefficients. We then no longer work in the space E (of large dimension), but in the space generated by the ϕ_i (which is a r-dimensional space). The gain in terms of reduction of the dimension is very large, since in practice one can take $r < 100$ and the gain in computation time is rather consequent.

Objectives

The main objective of this internship is to compare different reduction approaches, both deterministic and stochastic [1], and to evaluate their performance for sensitivity analysis. The models considered are parameterized partial differential equations, which can be considered as prototypes for more realistic environmental models (oceanographic, climate). We will provide error bars [2] for the estimation of sensitivity indices, as it is now a common requirement, particularly in environmental sciences.

Work outline

This internship will be balanced between the following three aspects:

- bibliography on the sensitivity analysis and the different reduction techniques;

- adaptation for a specific PDE model;
- implementation using softwares R and Matlab.

Useful skills

Statistics; R and / or Matlab; PDE and numerical analysis.

Date and place of the internship

The internship will take place in the Laboratory Jean Kuntzmann, in the MOISE Inria team (Clémentine Prieur and Maëlle Nodet). Optionally, the internship could take place in Orsay, Paris XI (Alexander Janon).

Continued PhD

This internship will eventually lead to a thesis in the MOISE team.

Gratification

Traditional for an internship (436 euros / month).

References

[1] S. Boyaval, C. Le Bris, T. Lelièvre, Y. Maday, N. C. Nguyen, and A. T. Patera. Reduced basis techniques for stochastic problems. *Arch. Comput. Methods Eng.*, 17(4):435–454, 2010.

[2] A. Janon, M. Nodet, and C. Prieur. Uncertainties assessment in global sensitivity indices estimation from metamodels. Preprint available at `http://hal.inria.fr/inria-00567977`, 2011, *Accepted in International Journal for Uncertainty Quantification.*

[3] Alexandre Janon, Thierry Klein, Agnes Lagnoux-Renaudie, Maëlle Nodet, and Clémentine Prieur. Asymptotic normality and efficiency of two Sobol index estimators. *ESAIM: Probability and Statistics*, 2013.

[4] A. Saltelli, M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, and S. Tarantola. *Global sensitivity analysis: the primer.* Wiley Online Library, 2008.

[5] I.M. Sobol. Global sensitivity indices for nonlinear mathematical models and their Monte Carlo estimates. *Mathematics and Computers in Simulation*, 55(1-3):271–280, 2001.

[6] J.Y. Tissot and C. Prieur. Variance-based sensitivity analysis using harmonic analysis. Technical report, `http://hal.inria.fr/hal-00680725`, 2012.