MSIAM2 MSc thesis project proposal:

Data Consistency and Nullspaces in Tomography from Circle Data

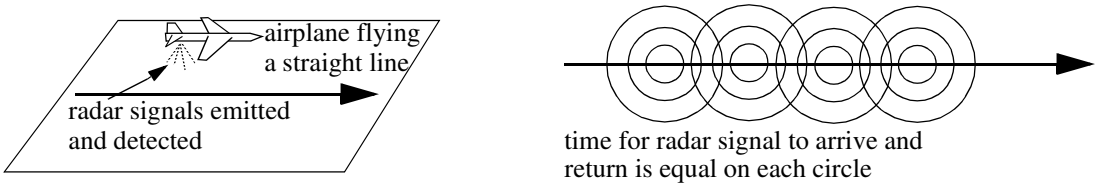
Supervisors: Rolf Clackdoyle (rolf.clackdoyle@univ-grenoble-alpes.fr)
Valérie Perrier (valerie.perrier@univ-grenoble-alpes.fr)
Laurent Desbat (laurent.desbat@univ-grenoble-alpes.fr)

Location and duration of Masters project: TIMC-IMAG laboratory. Five or six months.

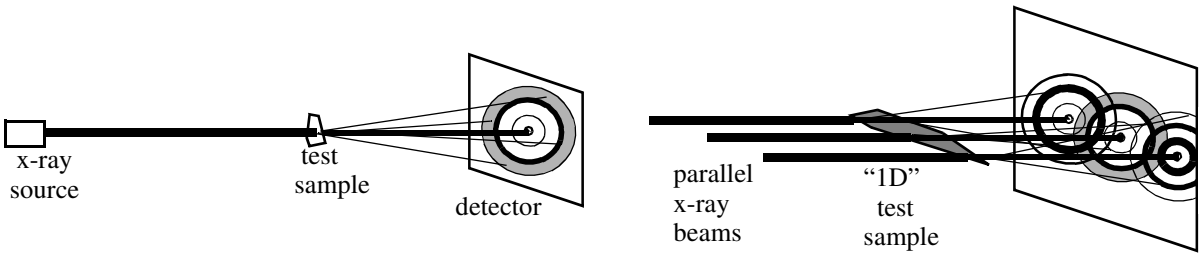
This is a research project, with the possibility of leading to a Ph.D. project.

Description:
Classical computed tomography deals with the mathematical problem of inverting the two-dimensional (2D) Radon transform. The two-dimensional Radon transform maps functions (of compact support) on R^2 to functions whose domain is the set of lines in the plane. The objective is to invert this transformation. In physical terms, all that is known about the 2D function is its integrals over straight lines. This problem and its solution is well-known, and is the basis of X-ray CT scanners that exist in every major hospital worldwide.

In this project, we consider the problem of recovering an unknown 2D function from its integrals over circles, whose centres all lie along a fixed line. There are also known inversion formulas for this problem. One application of this version of tomography is in Synthetic Aperture Radar (SAR) where an airplane flies along a straight line emitting radar signals whose return signals are sampled in time and therefore integrate landscape features along circles.



In another quite different imaging application, whereby scattered radiation from an x-ray probe is detector in circular rings on a detector [1], the mathematical forward problem is the dual of the SAR problem. This duality means that the nullspace of one problem corresponds to the consistency conditions [2] of the other space. Very little is known about the scatter imaging problem, so this project will use the much better understood SAR problem [3] to extract information about the scatter imaging problem. In particular, the consistency conditions of the SAR case will be used to explore the nullspace of the scatter problem.



Required skills: A solid knowledge of classical tomography is very strongly recommended. Students having successfully completed the course "Medical Imaging and Tomography" will have the necessary mathematics background. Some numerical experiments (computer simulations) will also be involved, so reasonable programming skill in python, or matlab, or C++, or IDL would be useful.

References:

[1] K.Landheer and P.Johns. "Synchrotron-based coherent scatter x-ray projection imaging using an array of monoenegetic pencil beams." *Rev. Sci. Instru.* **83**: 095114 (7 pages), 2012.
[2] R.Clackdoyle, L.Desbat, J.Lesaint, and S.Rit. "Data consistency conditions for cone-beam projections on a circular trajectory." *IEEE Sig Proc. Let.* **12**:1746-1750, 2016.
[3] L.-E. Andersson. "On the determination of a function from spherical averages." *SIAM J. Math. Anal.* **19**: 214-232, 1988.