Positivity certificates for polynomials using amœbas

Supervision: Victor Magron[1] Xavier Allamigeon[2] Stéphane Gaubert[2]
Contacts: victor.magron@imag.fr xavier.allamigeon@inria.fr stephane.gaubert@inria.fr
Homepage: http://www-verimag.imag.fr/~magron/
Working place: Laboratoire VERIMAG, Grenoble

Keywords polynomial optimization, certification, geometric programming, semidefinite programming, polynomial sum of squares, amœbas, simplex, circuit polynomials, entropy maximization, probability measures.

Context Finding positivity certificates for a multivariate polynomial is a central problem in optimization. The method providing more accurate certificates consists of finding a sum of squares decomposition of this polynomial, which boils down to solving a convex optimization problem (called a *semidefinite* program). Even though such certificates can be computed in polynomial time (up to an additive error), this framework only provides sufficient conditions: a bivariate nonnegative polynomial is not necessarily a polynomial sum of squares.

Iliman et al. [2] recently introduced a new type of positivity certificates, relying on Archimedean amœbas. The latter mathematical objects have been studied for fifteen years and are the images of polynomial system solutions under the log-module map. Amœbas have various combinatorial properties related to polytopes. The framework proposed in [2] combines these properties with convex optimization methods to compute certificates in a more efficient way. However, this approach only applies to a restricted class of polynomials, whose supports are simplex Newton polytopes.

Objectives We believe that this approach could be extended to more general polynomial optimization problems, leading to the following two-fold research investigation:

1. identifying polynomial systems for which the framework from [2] can be applied
2. implementing a tool to perform numerical experiments relying on geometric programming (e.g. with the cvx [1] Matlab toolbox)

Required Skills

- Convex optimisation, geometric and semidefinite programming
- Linear algebra basics
- Programming with OCaml/C/C++/Matlab according to preference of the candidate
- Related master courses: Software Development Tools and Methods, Efficient methods in optimization

A related PhD topic can be foreseen.

References

[1] M. Grant and S. Boyd. CVX: Matlab software for disciplined convex programming, version 2.1. http://cvxr.com/cvx, Mar. 2014.

[2] S. Iliman and T. de Wolff. Lower Bounds for Polynomials with Simplex Newton Polytopes Based on Geometric Programming. *ArXiv e-prints*, Feb. 2014.

[1]CNRS-VERIMAG Tempo team, Centre Equation 3, Office 15, 2 avenue de Vignate, 38610 Gières FRANCE
[2]INRIA Maxplus team, Centre de Mathématiques Appliquées de École Polytechnique, 91128 Palaiseau France