Intership Proposal MSIAM

Riesz and Radon Transforms

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Possibility of a PhD thesis: yes

Context and objectives In image processing, the Riesz transform was introduced as a possible extension of the Hilbert Transform in general dimension: it has many applications, like the extraction of local features in 2D signals, the demodulation of holograms, or the analysis of color images. The most easy way to implement the Riesz transform is to compute its Fourier representation by mean of 2D FFT. The disadvantage of such solution is first to require the whole knowledge of the 2D function, second to use a non local filter, equivalent of the ramp filter in 1D. Alternative approaches involve the Radon transform.

The Radon transform is widely used in medical imaging, for example in 2D tomography, modeling the X-ray attenuation through a patient in CT scanner. The conventional inversion of the Radon transform is the Filtered BackProjection (FBP) method, when all the data are available. It is well known that this inversion involves the Hilbert Transform, a non local operator, which leads to reconstruction errors in the case of truncated projection data [3].

To take advantage of the computational simplicity of the Radon transform to implement the Riesz transform, a first relation between the Riesz and the Radon transforms has been established in [2], and implemented using FFT in [4]. Unfortunately, this relation involve non local operators, mainly the Hilbert transform. This drawback was improved in the recent paper [1] by deriving a local formula for the computation of the Riesz transform, *locally* from *local* Radon data, in dimension 2 (and even dimensional spaces). This local approach has many advantages: first, from the numeric point of view, it involves less computations. For medical applications, local approaches allows for the use of small scanner detectors.

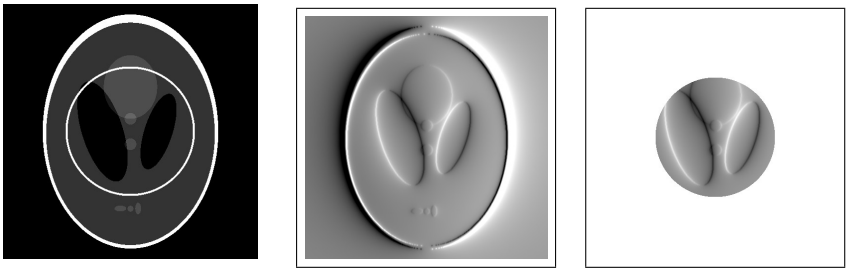

Figure 1: Shepp and Logan phantom, its Riesz transform (first component), from the full Radon data, and from its truncated (ROI) Radon data.

The aim of the following Master thesis project is extend the result of [1] in the three-dimensional case, by considering the X-ray transform (or the Cone beam transform) instead of the Radon transform, and the possibility to compute the Riesz transform from local projection data (Radon, X-ray, Cone Beam). The numerical tests will be conducted with MATLAB.

References

[1] L. Desbat and V. Perrier. On locality of Radon to Riesz transforms.

[2] M. Felsberg and G. Sommer. The monogenic signal. *IEEE Transactions on Signal Processing*, 49(12):3136–3144, 2001.

[3] F. Natterer. *The Mathematics of Computerized Tomography*. Wiley, 1986.

[4] R. Soulard and P. Carré. A discrete approach to monogenic analysis through Radon transform. In *Applied Geometric Algebras in Computer Science and Engineering (AGACSE 2012), La Rochelle, France*, 2012.