MSIAM M1: Variational Methods Applied to Modeling

Emmanuel Maitre (emmanuel.maitre@univ-grenoble-alpes.fr)
Clément Jourdana (clement.jourdana@univ-grenoble-alpes.fr)

Introduction:

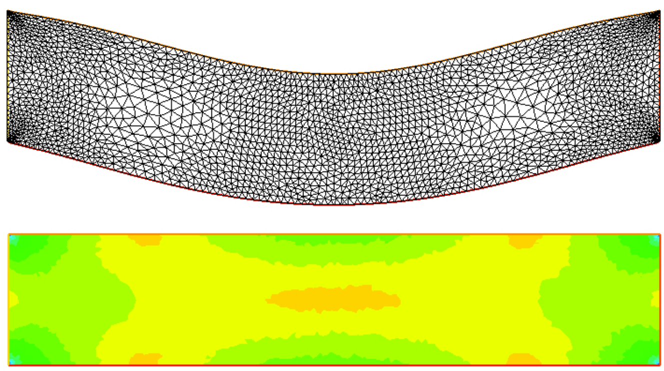
During the first semester, you discussed the finite difference method for PDE resolution. A major disadvantage of this approach is that it is difficult to apply it on non rectangular domains since it relies on a cartesian grid. This method works directly on a discretization of the equation via a discretization of the domain. In this course, we introduce the finite element method, that is slightly different and more abstract. It consists in discretizing the space of solutions rather than the domain on which the equation is posed (although to discretize the functional space, we will use a discretization of the domain). This type of approach allows to deal with coupled phenomena of great complexity that one encounters in engineering. The aim of this course is to become familiar with the finite element method and to study its application to some modeling problems.

Content:

I. Boundary problems and variational forms. Sobolev spaces.
II. Stationary models / Elliptic equations (variational framework, symmetric case and link to minimization, green formulas).
III. Finite element method (basis functions, algorithms, implementation, a priori error analysis).
IV. Introduction to modeling from few examples: thermal, transport, elasticity (Lamé), fluids (Stokes), fluid-structure coupling (flow around an elastic obstacle)...
V. Non stationary models / Parabolic equations (time schemes, splitting methods).
VI. Possible complements: nonlinear case and linearization, ALE method for fluid-structure coupling, model reduction, discontinuous Galerkin, a posteriori analysis and mesh refinement.

Practicals:

The practical aim is to get familiar with FreeFem++, an environment that allows to easily solve PDEs in dimension 2 using a finite element method. Different modeling examples (see item IV above) will be considered and we will discuss the mathematical difficulties specific to each physical phenomenon.



*Refined mesh of an embedded beam deformed by a vertical force (top)
and norm of the corresponding stress tensor (bottom) obtained using FreeFem++.*