INTERNSHIP PROPOSAL
MASTER OF SCIENCE FOR INDUSTRIAL AND APPLIED MATHEMATICS
2014–2015

Asymptotic preserving schemes for Maxwell–Bloch equations

Theme: numerical modelling, partial differential equations.

Location: Laboratoire Jean Kuntzmann, Grenoble.

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Summary

Maxwell–Bloch system models the interaction of an electromagnetic field (laser) with matter described at the quantum level, which in particular yield the probability of matter electrons to be in a specific quantum state. This system couples Maxwell equation (a partial differential equation) with a collection of ordinary time-differential equations at each space point [1]. Averaging techniques in the case of a small incident electric field (of size ε) allow to derive an asymptotic model that involve an equation over long times for the field (nonlinear Schrödinger equation) with a lower dimensional set of ordinary differential equations (rate equations).

When the original system is discretized by classical methods, there is usually no reason why numerical solutions should tend to solutions of the asymptotic model as $\varepsilon \to 0$. On the contrary asymptotic preserving schemes allow this [2].

$$\begin{array}{ccc} P^{\varepsilon,h} & \xrightarrow{\quad h \to 0 \quad} & P^{\varepsilon} \\ \Big\downarrow \varepsilon \to 0 & & \Big\downarrow \varepsilon \to 0 \\ P^{0,h} & \xrightarrow{\quad h \to 0 \quad} & P^{0} \end{array}$$

The goal of this internship is to begin to explore the track of asymptotic preserving schemes (initially developed for other equations) to Maxwell–Bloch equations. This internship could be continued in a PhD thesis on a University research grant.

Bibliography

[1] Brigitte BIDÉGARAY-FESQUET. *Hiérarchie de modèles en optique quantique. De Maxwell–Bloch à Schrödinger non linéaire.* XIV+175 pages, Collection Mathématiques et Applications, volume 49, Springer (2006).

[2] SHI Jin, *Efficient asymptotic-preserving (AP) schemes for some multiscale kinetic equations*, SIAM Journal on Scientific Computing, **21**, 441–454 (1999).